SM3 HW 14.3 Z-Scores and the Standard Normal Distribution

14.3 EXERCISES **NAME**_____

You have to make a final decision on two applicants. They are both wonderful students with the very same G.P.A. and class rankings. It all comes down to their test scores. Student A took the ACT and received a score of 29 in mathematics. Student B took the SAT and received a score of 680 in mathematics. Since you are an expert in college entrance exams, you know that both tests are designed to be normally distributed. A perfect ACT is 36. The ACT mathematics section has a mean of 21 and standard deviation of 5.3. A perfect score on the SAT math section is 800. The SAT mathematics section has a mean of 516 and a standard deviation of 116.

1. What was Student A's z-score? **1.51**
2. What was Student B's z-score? **1.41**
3. Which student would you choose? **Student A** Why? **He tested higher (higher z-score) relative to those that took the same test than Student B did.**
4. What is the probability that a student scored better than Student A on the ACT test? **normalcdf(1.51,1E99,0,1)=0.0655**
5. What proportion of students would have z-scores that fell between Student A and Student B? **normalcdf(1.41,1.51,0,1)=0.0137**

Let's say you had two imaginary test takers, Jack and Jill. Jack's z-score was 1.49 and Jill's z-score was 0.89.

6. What percent of the test takers scored below Jack? **normalcdf(-1E99,1.49,0,1)=0.9319** What percent scored above Jack? **normalcdf(1.49,1E99, 0,1)=0.0681 or 1-0.9319 = 0.0681 (complement of first question)**
7. What percent of the test takers scored below Jill? **normalcdf(-1E99, .89,0,1)=0.8133** What percent scored above Jill? **normalcdf(.89,1E99, 0,1)=0.1867 or 1-0.8133 = 0.1867 (complement of first question)**
8. What percent of the test takers scored between Jack and Jill? **normalcdf(0.89, 1.49, 0,1)=0.1186**
9. Jack and Jill's friend, Jason, scored -1.49. Find the number of test takers that scored above him without using a table or technology. Explain your strategy. **Greater than the negative Z-score will give the same probability as less than the positive z-score. So Jason has the opposite z-score as Jack so the probability will be the same as Question #6 which is 0.9319**

At South Beach High School, there are 2500 students attending. Mariana surveys 40 of her friends where they prefer to eat lunch. She created the following two---way table showing her results:

	9th Grade	10th Grade	11th Grade	12th Grade	Totals
School Cafeteria	18	6	2	1	**28**
Off Campus	2	4	3	4	**12**
Totals	**20**	**10**	**5**	**5**	**40**

Mariana plans to use her data to answer the following questions:

I. Do students prefer to eat on campus or off campus overall?

II. Is there a difference between grade levels for where students prefer to eat lunch?

10. In Mariana's sample, what percent of students prefer school lunch? **70%** What percent prefer to eat off campus? **30%**

11. For each grade level in her sample, determine the percent of students that prefer school lunch and the percent that prefer off campus lunch. Do you notice anything unusual?

    9th Grade: 90% School 10% Off Campus **10th Grade: 60% School 40% Off Campus**

    11th Grade: 40% School 60% Off Campus **12th Grade: 20% School 80% Off**

12. Based on her sample, Mariana concludes that students at South Beach High School overall prefer school lunch. Do you agree or disagree? **Disagree** Why? **Half of the sample was taken from the 9th graders who can't drive yet and probably eat on campus most of the time and don't have the option to eat off campus. The grades that could have driver's licenses favor eating off campus much more than under classmen. If the sample was taken equally from each class it would have given us a much better understanding of the population.**

A company makes a mean monthly income of $20,300 with a standard deviation of $3,200. In one given month the company makes $29,500.

13. Find the z-score. **z=2.875**

14. Assuming the companies monthly income is Normal, what percent of the time does the company make more than this amount? Less than? **0.2% of the time it makes more than $29500 in one month. 99.8% of the time it makes less than $29500 in one month**

15. What percent of the time does the company make between $15,000 and $25,000? **normalcdf(15000,25000,20300,3200)=88.02%**

16. If the company needs to make $16,400 in order to break even, how likely in a given month is the company to make a profit? **Normalcdf(16400, 1E99, 20300,3200) = 88.9%**

On the Wechsler Adult Intelligence Scale, an average IQ is 100 with a standard deviation of 15 units.

17. IQ scores between 90 and 109 are considered average. Assuming IQ scores follow a Normal distribution, what percent of people are considered average? **Normalcdf(90,109,100,15) = 47.3%**

18. One measure of Genius is an IQ score of above 135. What percent of people are considered genius? **Normalcdf(135, 1E99,100,15) = 0.9%**

19. Einstein had an IQ score of 160. What is his z-score? **z = 4**

20. What is the probability of an individual having a higher IQ than Einstein? **normalcdf(4, 1E99, 0, 1) = 0.0000317 or normalcdf(160, 1E99, 100, 15) = 0.0000317, not too likely!**